IS THE NUMBER OF PRIMES $\frac{1}{2}\sum_{i=0}^{n-1} i!$ FINITE ?

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For $n \geq 1$ let

$$!n = \sum_{i=0}^{n-1} i! \ .$$

$!2 \equiv 0 \pmod 2$ then $!n$ is divisible by 2 for all $n \geq 2$. The numbers $!n/2$ are prime for $n = 4, 5, 8, 9, 10, 11, 30, 76, 163, 271, 273, 354$ and no other $n < 500$. Are there infinitely many prime numbers of the form $!n/2$? If it is true then $p \nmid !p$ for all prime numbers $p > 2$. Let

$$r_p = !p \bmod p$$

Miodrag Zivković extended the computation of the r_p and verified that $r_p \neq 0$ for $2 < p < 2^{23}$ [1].

A program was written by the author to extend this computation. The inner loop of the program consists in the simple algorithm

$$\begin{aligned} f &= (f \times i) \mod p \\ s &= (s + f) \mod p \\ i &= i + 1 \end{aligned} \tag{1}$$

The floating-point unit is used to replace the slow integer division by a multiplication by $1/p$. Because the internal precision of the FP-registers is 64 bits, a value is rounded to the nearest integral value by adding and subtracting the constant $2^{63} + 2^{62}$. The value of s is accumulated without modular reduction and only one modular reduction is evaluated outside of the loop. Finally, two different p are checked simultaneously in the same loop to improve the pipelining of the floating-point instructions. With these optimizations, the computation time of Alg.1 is 14 cycles on a PII/Celeron/PIII processor.

r_p was computed by the program for $2 \leq p < 2^{26} = 67108864$: the results of [1, Table 1] were verified and one new r_p such that $|r_p| < 10$ was found: $r_{11477429} = 9$. No solution to $r_p = 0$ was discovered, then the question of whether the number of primes $\frac{1}{2}\sum_{i=0}^{n-1} i!$ is finite remains an open question.

References

1. M. Zivković, *The number of primes $\sum_{i=1}^{n}(-1)^{n-i}i!$ is finite*, Math. Comp. **68** (1999), 403–409.

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